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DYNAMIC CHARACTERIZATION OF NOISE AND VIBRATION TRANSMISSION PATHS IN LINEAR CYCLIC SYSTEMS: PART I — THEORY

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Abstract

Linear cyclic systems (LCS) are a class of systems whose dynamic behaviors change cyclically. The understanding of noise and vibration transmission paths in LCS is quite limited due to the time-varying nature of their dynamics. The first part of this two-part paper derives a generic expression that describes how the noise and/or vibration are transmitted between two (or multiple) locations in LCS. The cyclic nature of LCS' transfer functions is shown to generate a series of amplitude modulated input signals whose carrier frequencies are harmonic multiples of the LCS' fundamental frequency. Applicability of signal processing techniques for linear time-invariant systems (LTIS) to general LCS is also discussed. Then, a criterion is proposed to determine how well LCS can be approximated as LTIS. In Part II, some experimental results validate the analyses carried out in Part I.

1 INTRODUCTION

Dynamic systems with fundamental repetitive motions frequently exhibit cyclic behaviors and consequently generate periodic mechanical vibration and acoustic radiation. In most cases, the noise and vibration emanating from these systems turn out to be nuisance, which need to be either eliminated or at least reduced before they are put into practice. Considering the importance of controlling noise and vibration in cyclic systems (CS), it is not surprising that numerous papers have addressed the various issues related to this subject such as noise and vibration signal analysis [1, 2, 3, 4], noise and vibration control [5], transmission path identification [6, 7, 4], etc., where only recent papers have been listed. These earlier works can be categorized into two groups by their assumptions on the transmission path dynamics:

1. The transmission path dynamics is linear time-invariant (LTI).
2. The transmission path dynamics is irrelevant to the problem under consideration.

The above assessment naturally motivates us to analyze the transmission paths of noise and vibration in CS without *a priori* assuming that the transmission paths are LTIS. The resulting transmission path analysis may play a role in validating the assumption that the transmission paths are LTIS and understanding how the input signals are related to the output signals, which obviously helps to understand the input/output signals themselves.

Multiplicative behavior of the generated mechanical vibration/acoustic radiation and the transmission paths (both cyclically time-varying) does not seem to allow such general cyclic system (CS) to be readily analyzable, which partially explains lack of understanding the signals and transmission paths in general CS. On the other hand linear cyclic systems (LCS) are a subclass of general CS, whose transmission media are linear and cyclic. Their relatively simple and analytically tractable dynamics makes amenable the analysis of LCS' signals and transmission paths. Once the scope of this paper is confined to LCS, the linearity of the transmission paths makes it possible to represent the output signals as a convolution integral of the input signals and (cyclically) time-varying impulse response functions [8]. Then, via Fourier transform and Fourier series analyses, the convolution integral is further simplified as an input-output transfer function, from which various subsequent analyses are carried out. LCS are classified into two groups based on the relative magnitudes of the carriers: genuine linear cyclic systems (GLCS) and pseudo linear cyclic systems (PLCS). A subsequent analysis leads to a criterion to determine how well LCS can be approximated as LTIS. PLCS can be well approximated as LTIS, while GLCS cannot.

Section 2 derives the expression describing the input/output relation of an LCS, where the Fourier transform and Fourier series analyses provide two indispensable tools. Section 3 explains the behaviors of GLCS and the applicability of the signal processing techniques for LTIS to GLCS. Section 4 shows the analysis for PLCS, similar to that in Section 3. A criterion to determine how well LCS can be approximated as LTIS is also proposed in Section 5.

2 INPUT/OUTPUT RELATION OF LINEAR CYCLIC SYSTEMS

In this section, a mathematical description of noise and vibration transmission path in a simple LCS is derived. Figure 1 shows the schematic of an LCS that consists of three components: inner sphere, rotating ellipse, outer spherical shell. The system is simple but captures the essence of the LCS in terms of noise and vibration transmission, which is required for the analysis in this paper to be valid for the general, more complex LCS. Assume that the system (or to be specific, the ellipse) is running at the angular frequency ω . Define an abstract entity (or angle) θ as an indicator of the system status during the repetitive (or cyclic) motion. As the system goes through a full cycle, θ increases monotonically from 0° to 360° . The transducers 'A' and 'B' in Figure 1 are used either to instrument the LCS or to excite the LCS. The transducers 'A' and 'B' are located externally on the shell and internally on the inner sphere, respectively, which are the fixed points of the LCS. Specific requirements for analysis determine the types of transducers. The external transducer is used as the input transducer throughout the analysis in this section for simplicity. Throughout this paper, the distinction between the forward path (from 'B' to 'A') and reverse path (from 'A' to 'B') in Figure 1 is not made explicitly unless specified otherwise, since the most analyses are regardless of the path direction.

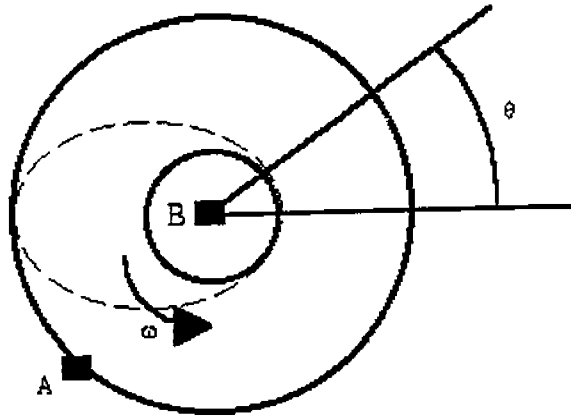


Figure 1: The schematic of a linear cyclic system

We start the analysis of an LCS by defining its impulse response. Assume that the LCS in Figure 1 is running in the steady state. While the LCS running, an impulse is applied at the external transducer 'A' (a force transducer) at the angle θ_0 . In the meantime, the time history of the acceleration measurement at the internal transducer 'B' (an accelerometer) is recorded. Then, the recorded time history is defined as the impulse response of the LCS, $h(\theta_0, t)$, at θ_0 . θ_0 is the system status angle when the impulse is applied at the force transducer 'A'. The second argument t is the elapsed time from the instant when the impulse is applied.

Once the impulse response is defined, it is straightforward to express the input/output relation for an LCS. Denote the input and output at time t as $u(t)$, $y(t)$ respectively. Then $y(t)$ becomes [8]

$$y(t) = \int_{-\infty}^t h(\theta(\tau), t - \tau) u(\tau) d\tau, \quad (1)$$

where $\theta(t) = \theta_0 + 2\pi \int_0^t f_r(t') dt'$ and $f_r(t)$ is the instantaneous frequency of the LCS at time t . The argument t is dropped for simplicity in the following unless necessary for clarification. $\theta(t)$ is periodic with the fundamental period T_R . Also define $f_R (= 1/T_R)$ as the average (or equivalent) fundamental frequency.

Taking Fourier Transform (FT) of (1) yields

$$Y(f) \stackrel{\text{def}}{=} \mathcal{F}[y(t)] = \int_{-\infty}^{\infty} \int_{-\infty}^t h(\theta(\tau), t - \tau) u(\tau) d\tau \exp(-j2\pi f t) dt. \quad (2)$$

Introducing a new variable $p = t - \tau$ gives

$$Y(f) = \int_{-\infty}^{\infty} \int_0^{\infty} h(\theta(\tau), p) \exp(-j2\pi fp) dp u(\tau) \exp(-j2\pi f\tau) d\tau = \int_{-\infty}^{\infty} H(\theta(\tau), f) u(\tau) \exp(-j2\pi f\tau) d\tau. \quad (3)$$

$$H(\theta(\tau), f) \stackrel{\text{def}}{=} \int_0^{\infty} h(\theta(\tau), p) \exp(-j2\pi fp) dp = \int_{-\infty}^{\infty} h(\theta(\tau), p) \exp(-j2\pi fp) dp,$$

where the last equality comes from the causality of $h(\theta, t)$. Then, the periodicity of $H(\theta, f)$ with respect to its first argument θ (with the fundamental period T_R) allows the following Fourier expansion (with respect to θ):

$$H(\theta, f) = H(\theta(\tau), f) = \sum_{k=-\infty}^{\infty} A_k(f) \exp(j2\pi k f_R \tau), \quad (4)$$

$$A_k(f) = \int_{-\pi/T_R}^{\pi/T_R} H(\theta(q), f) \exp(-j2\pi k f_R q) dq, \quad (5)$$

where $A_k(f)$ is the Fourier coefficient corresponding to the k^{th} harmonic at the frequency f .

Equations (4) and (5) turn out to be essential to the analyses of two representative classes of LCS described in Sections 3 and 4. The characteristics of LCS depend heavily upon the magnitude of $A_k(f)$. Two extreme but still representative cases are considered in this paper:

1. $A_k(f)$ are significant for some $k(\neq 0)$ at some given frequency f .
2. $A_k(f)$ are relatively small for all $k(\neq 0)$ at any given frequency f .

The corresponding systems may be classified as a genuine LCS and a pseudo LCS, respectively. Above rather ad-hoc criterion is more rigorously presented in Section 5.

3 GENUINE LINEAR CYCLIC SYSTEMS

We first examine the transfer function $H(\theta, f)$ itself for a given f . The genuine linear cyclic systems (GLCS) have been defined such that there exist some significant $A_k(f)$ for some frequency f , which implies that there exists significant periodic fluctuation of transfer function $H(\theta, f)$. The number of significant $A_k(f)$ ($k \neq 0$) and k together determine how the transfer function $H(\theta, f)$ fluctuates over a cycle at a given frequency f .

Now the effect of the cyclically time-varying transfer function $H(\theta, f)$ of GLCS on the input/output relation is evaluated. Substituting (4) into (3) yields

$$Y(f) = \sum_{k=-\infty}^{\infty} A_k(f) \int_{-\infty}^{\infty} u(\tau) \exp[-j2\pi(f - kf_R)\tau] d\tau. \quad (6)$$

For simplicity, only three terms for $k = -1, 0, 1$ are assumed to be significant. Then (6) can be readily interpreted as the $A_k(f)$ -weighted sum of the FT of three amplitude modulated (AM) signals with the input signal $u(\tau)$ and the carrier frequencies $-f_R, 0, f_R$. By modulation theorem of the FT, the spectrum of $Y(f)$ has three $A_k(f)$ -weighted replicas of the baseband spectrum of $U(f) [= \mathcal{F}(u(\cdot))]$, shifted by $-f_R, 0, f_R$ [8]. This result obviously applies to a GLCS excited by a single sinusoid input with unit amplitude at the frequency f_e . Since the modulation property is regardless of the excitation input, the input sinusoid at the frequency f_e is modulated with the carriers at the frequencies $-f_R, 0, f_R$. As a result, three sinusoids with amplitudes $A_{-1}(f_e - f_R), A_0(f_e), A_1(f_e + f_R)$ at the frequencies $f_e - f_R, f_e, f_e + f_R$ are observed at the output. Generating three frequency shifted replicas of the input spectrum is not a behavior of an LTIS.

Up until now, a clear distinction has not been made between two transmission paths: 1) the forward path along which the LCS generated noise (at the transducer 'B') is transmitted to the shell (at the transducer 'A') 2) the reverse path along which an externally excited signal ('A') is transmitted to the internal transducer ('B'). Now it remains to examine distinctive features of each path and to show whether two paths are equivalent or the transfer functions along two paths are identical. The equivalence of two paths is particularly important when the forward

path cannot be readily identified, while the reverse path may be identified, e.g. through the reciprocity principle. Remember that the system under consideration has to be LTI and passive in order for the reciprocity principle to be applicable [9].

The reverse path is first analyzed due to its simplicity. In the reverse path, only one excitation frequency exists. Three amplitude modulated sinusoids would be observed at the output. Here these sinusoids are examined in detail. Exciting a GLCS with a sinusoid at the frequency f_e would give

$$Y(f) = \sum_{k=-1}^1 A_k(f) \int_{-\infty}^{\infty} \exp[j2\pi f_e(\tau + t_0)] \exp(j2\pi k f_R \tau) \exp(-j2\pi f \tau) d\tau, \quad (7)$$

where t_0 is the time lag of the input sinusoid relative to the reference of the system cyclic behavior (e.g. crankcase angle in a certain rotating machine). With the input sinusoid as a reference, (7) becomes

$$Y(f) = \sum_{k=-1}^1 A_k(f, t_0) \int_{-\infty}^{\infty} \exp[j2\pi(f_e + k f_R - f)\tau] d\tau, \quad (8)$$

where

$$A_k(f, t_0) = \int_{-\pi/T_R}^{\pi/T_R} H(\theta(q), f) \exp(-j2\pi k f_R q) dq \exp(j2\pi k f_R t_0), \quad (9)$$

where the periodicity of $H(\theta, f) \exp(-j2\pi k f_R q)$ with respect to q was used in the derivation. Note that $A_k(f)$ depend on the time lag t_0 for $k \neq 0$. However, for $k = 0$, $A_k(f, t_0)$ does not depend on t_0 . For $k \neq 0$, the magnitudes of $A_k(f, t_0)$ do not change but the phases change as $\exp(j2\pi k f_R t_0)$. This can be summarized as follows: when a sinusoid is applied at the frequency f_e at several *randomly-selected* time instants t_0 , time invariant $A_0(t_0, f_e)$ is observed at f_e but time-varying $A_k(t_0, f_e + k f_R)$ at $f_e + k f_R$, for $k \neq 0$. This is experimentally demonstrated later in Part II of this two-part paper as a partial validation of our analysis.

Then, the forward path is analyzed. A GLCS generates internal noise with multiple harmonics of the revolution frequency f_R through various mechanisms, which is in turn transmitted to the external transducer through a GLCS which modulates the generated internal noise, where the carrier frequencies are harmonic multiples of f_R . This implies that multiple sinusoids at different frequencies may contribute to the output at a given frequency f , which leads us to the conclusion that the GLCS is not an LTIS.

From the discussion above, two paths are fundamentally different. It implies that signal processing techniques based on the assumption that the GLCS is an LTIS would not give any meaningful answer.

4 PSEUDO LINEAR CYCLIC SYSTEMS

The pseudo linear cyclic systems (PLCS) have been defined such that $A_k(f)$ are relatively small for all k except $k = 0$ at any frequency f . This means that the variation of the transfer function over a cycle at a given frequency f is minimal.

The mathematical analysis starts with (3) and (4). Since $A_k(f)$ are relatively small, (4) can be approximated as

$$H(\theta, f) = H(\theta(\tau), f) \approx A_0(f) = \int_{-\pi/T_R}^{\pi/T_R} H(\theta(\tau), f) d\tau, \quad (10)$$

where only the mean value of $H(\theta(\tau), f)$ over a cycle is taken to approximate $H(\theta(\tau), f)$. Then, (3) becomes

$$Y(f) \approx \int_{-\infty}^{\infty} A_0(f) u(\tau) \exp(-j2\pi f \tau) d\tau = A_0(f) U(f), \quad (11)$$

where $U(f) = \int_{-\infty}^{\infty} u(\tau) \exp(-j2\pi f \tau) d\tau$. This is nothing but the input-output expression of a linear time invariant system (LTIS) in the frequency domain. There is no amplitude modulation as in the case of the GLCS. A PLCS is essentially an LTIS.

Once the behavior of a PLCS is identified as that of an LTIS, the behavior of a PLCS at one given frequency f_e can be readily predicted. When excited by a single sinusoid at f_e , a PLCS must have only one sinusoid as a response

at the excitation frequency f_e . The transfer function at f_e is the mean value of $H(\theta(\tau), f_e)$. Since f_e was arbitrary, this implies that the error would not be significant even if $H(\theta(\tau), f)$ for any τ is approximated as $H(\theta(\tau_s), f)$ at a specific time instant τ_s for all f .

The simple dynamic characteristic of a PLCS makes it dispensable to analyze the forward and reverse paths respectively. Then, it remains to show that the forward and reverse paths have the identical transfer functions or the impulse responses. Independence of the transfer functions (or impulse responses) with respect to cyclic positions effectively eliminates the cyclic behavior of the PLCS, which makes a PLCS an LTIS. Most mechanical systems are passive in that they do not increase the noise/vibration energy while transmitting the noise/vibration from one point to another; most mechanical systems dissipate or at most maintain the vibration energy. In this paper, it is assumed without proof that the systems under consideration belong to linear, passive systems. It is well-known that the reciprocity principle holds for a passive LTIS [9]. In this respect, the transfer functions for the forward and reverse paths in a PLCS are identical, which has the following important implication: the sideband criterion presented in Section 5 to classify an LCS into a GLCS or PLCS can be applied to the reverse path instead of forward path.

Finally, we discuss the applicability of the signal processing techniques to a PLCS. Since the PLCS is essentially an LTIS, any signal processing technique applicable to an LTIS (e.g. autoregressive modeling in [7]) can be applied to PLCS. In addition, since most mechanical systems (which is of our primary interest in this paper) are passive, signal processing techniques for passive LTIS should be applicable here.

5 THE SIDEBAND CRITERION FOR LCS

This section presents a criterion to determine how well an LCS can be approximated as an LTIS: the sideband criterion for LCS

Given an LCS, its impulse response $h(\theta, t)$ and a pre-determined critical number α , the LCS can well be approximated as an LTIS if the following condition holds:

$$\sup_{\theta \in [0^\circ, 360^\circ]} \int_{-\infty}^{\infty} |h(\theta, t) - \text{avg}_\theta h(\vartheta, t)|^2 dt < \alpha \text{avg}_\theta \int_{-\infty}^{\infty} |h(\theta, t)|^2 dt. \quad (12)$$

If the Fourier transform of $h(\theta, t)$, $H(\theta, f)$, is given instead, the above relation (12) can be equivalently expressed in the frequency domain (using Parseval's theorem [8]) as:

$$\sup_{\theta \in [0^\circ, 360^\circ]} \int_{-\infty}^{\infty} |H(\theta, f) - \text{avg}_\theta H(\vartheta, f)|^2 df < \alpha \text{avg}_\theta \int_{-\infty}^{\infty} |H(\theta, f)|^2 df. \quad (13)$$

$\sup_x \Psi(x)$ denotes the supremum or maximum of the function Ψ over its argument(s) x . $\text{avg}_x \Phi(x)$ is the mean value of the function Φ over its argument x . The critical number α is determined *a priori*. A guideline on choosing α is provided later in this section. Both (12) and (13) state that the LCS is an PLCS if the differential energy of the impulse response (or its Fourier transform in the frequency domain) varies less than $\alpha \cdot$ (its average energy) with respect to θ .

Although compact, the above criteria (12) and (13) are not trivial to evaluate. (4) allows us to derive a simpler criterion than (13) (refer to Section 3 to see how readily $A_k(f)$ can be obtained). First, the left-hand side of (13) is bounded above as follows:

$$\sup_{\theta} \int_{-\infty}^{\infty} |H(\theta, f) - \text{avg}_\theta H(\vartheta, f)|^2 df \leq \int_{-\infty}^{\infty} [\sum_{k \neq 0} |A_k(f)|]^2 df. \quad (14)$$

Similarly, the right-hand side of (13) can be expressed in terms of $A_k(f)$. Changing the order of two operators "avg" and "∫" gives

$$\text{avg}_\theta \int_{-\infty}^{\infty} |H(\theta, f)|^2 df = \int_{-\infty}^{\infty} \sum_{k=-\infty}^{\infty} |A_k(f)|^2 df, \quad (15)$$

where the "avg" operator sifts only DC terms to derive the last equality. With (14) and (15), (13) can be transformed into the following:

$$\int_{-\infty}^{\infty} [\sum_{k \neq 0} |A_k(f)|]^2 df < \alpha \int_{-\infty}^{\infty} \sum_{k=-\infty}^{\infty} |A_k(f)|^2 df. \quad (16)$$

It is worth noting that (13) and (16) are not equivalent (see (14)). It is possible that (13) ("tight") may be satisfied, while (16) ("loose") is not. However, it is much simpler to evaluate (16) than (13). We propose to adjust the critical number α to achieve an optimal tradeoff between the tightness (of the bound) and simplicity. Earlier in this section when the sideband criterion is presented, the critical number α is a number given *a priori*. From (12), (13) and (16), we can conclude that α can be interpreted as a quotient between the variation energy and average energy. There exist no general rules that specify the optimal α and the associated risk factor to take into account the looseness of (16) but our experience shows that the following rule works effectively though heuristic: 1) select α based on *a priori* knowledge. 2) multiply α by the risk factor 1.5.

The sideband criterion in this paper renders some nice features, among which it stands out that the criterion is evaluated in the frequency domain rather than in the time domain. Due to harmonic disturbance from the running machine, sensor/actuator noise, etc., it is very difficult to obtain disturbance-free data in the time domain. The frequency domain experiment can avoid the harmonic disturbance by probing at the frequencies between harmonics. In addition, experiments in the frequency domain provide certain advantages over those in the time domain. First of all, the effective signal-to-noise ratio within the frequency range of interest can be significantly improved. Secondly, the higher signal-to-noise ratio for a given input excitation energy makes it possible to prevent any possible nonlinearity by lowering the level of the input signal.

6 CONCLUDING REMARKS

A generic expression for analyzing LCS is derived. The cyclic nature of LCS is shown to generate a series of amplitude-modulated signals whose carrier frequencies are harmonic multiples of the fundamental frequency of the LCS. A criterion is developed to classify an LCS into GLCS or PLCS. The criterion provides a simple experimental test, from which an LCS can be classified. The criterion can be checked before signal processing techniques for LTIS are applied to analyze rotating machinery, in order to validate the assumption that the dynamic system under consideration is LTI.

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